MATHEMATICS

AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE
(MAIN + ADVANCE) & COMPETITIVE EXAM.
FOR XI (PQRS)

DERIVATIVES
& Their Properties

CONTENTS

THINGS TO REMEMBER

1. A function f(x) is differentiable at x = c iff $\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ exists finitely.

   This limit is called the derivative or differentiation of f(x) at x = c and is denoted by f' (c).

2. Geometrically the derivative of a function f(x) at a point x = c is the slope of the tangent to the curve y = f(x) at the point (c, f (c)).

3. If f(x) is a differentiable function, then $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ is called the differentiation of f(x) or differentiation of f(x) with respect to x.

4. Mechanically, $\dfrac{d}{dx}$ (f(x)) measures the rate of change of f(x) with respect to x.

5. Following are some standard derivatives :

   (i) $\dfrac{d}{dx}(x^n) = n\, x^{n-1}$

   (ii) $\dfrac{d}{dx}(a^x) = a^x \log_e a,\ a > 0,\ a \neq 1$

   (iii) $\dfrac{d}{dx}(e^x) = e^x$

   (iv) $\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

   (v) $\dfrac{d}{dx}(\sin x) = \cos x$

   (vi) $\dfrac{d}{dx}(\cos x) = -\sin x$

   (vii) $\dfrac{d}{dx}(\tan x) = \sec^2 x$

   (viii) $\dfrac{d}{dx}(\cot x) = -\mathrm{cosec}^2\, x$

   (ix) $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

   (x) $\dfrac{d}{dx}(\mathrm{cosec}\ x) = -\mathrm{cosec}\ x \cot x$

6. Following are the fundamental rules for differentiation :

(i) Differentiation of a constant function is zero i.e., $\frac{d}{dx}(c) = 0$

(ii) Differentiation of a constant and a function is equal to constant times the differentiation of the function.

(iii) If f(x) and g(x) are differentiation function, then

(a) $\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

(b) $\frac{d}{dx}\{f(x) \times g(x)\} = \frac{d}{dx}(f(x)) \times g(x) + f(x) \times \frac{d}{dx}(g(x))$

(c) $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x) \times \frac{d}{dx}(f(x)) - f(x) \times \frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$

EXERCISE-1

1. Let f be a real valued function defined by $f(x) = x^2 + 1$. Find $f'(2)$.

2. Find the derivative of $f(x) = 2x^2 + 3x - 5$ at $x = -1$. Also, prove that $f'(0) + 3f'(-1) = 0$

3. If $f(x) = x^n$, where $n \in R$, then, the differentiation of x^n with respect to x is nx^{n-1}.

4. The differentiation of $\log_e x$, $x > 0$ is $\frac{1}{x}$

5. The differentiation of $\log_a x$ $(a > 0, a \neq 1)$ with respect to x is $\frac{1}{x \log_e a}$

6. The differentiation of tan x with respect to x is $\sec^2 x$.

7. Differentiation the following function with respect to x from first principals :

(i) $\sqrt{x}$

(ii) $\sqrt{ax+b}$

(iii) $\frac{1}{ax+b}$

8. Differentiate xe^x from first principle.

9. Differentiate the following function w.r.t. x from first principles : $\tan\sqrt{x}$

10. Differentiate $x^2 \cos x$ from first principle.

11. Differentiate $e^{\sqrt{\tan x}}$ from first principle.

12. Differentiate the following function with respect to x :

(i) $\log_x x$

(ii) $e^{3\log x}$

(iii) $9.(3^x)$

13.   If $y = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + ...,$ show that $\dfrac{dy}{dx} = y$.

14. $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1} x + a_n$.

15.   Find the rate at which the function $f(x) = x^4 - 2x^3 + 3x^2 + x + 5$ changes with respect to x.

16.   Differentiate the following functions with respect to x : x sin x

17.   Differentiate the following functions w.r.t. x :

   (i) $x^3 e^x \sin x$

   (ii) $x^n \log_a x\ e^x$

18.   Differentiate the following functions w.r.t. x : $e^x \log \sqrt{x} \tan x$

19.   Differentiate the following functions w.r.t. x : $x^5 (3 - 6x^{-9})$

20.   Differentiate the following functions with respect to x : $\dfrac{ax^2 + bx + c}{px^2 + qx + r}$

21.   Differentiate the following functions with respect to x : $\dfrac{10^x}{\sin x}$

22.   Differentiate the following functions with respect to x : $\dfrac{1 + 3^x}{1 - 3^x}$

23.   Differentiate the following functions with respect to x : $\dfrac{x + \cos x}{\tan x}$

24.   Differentiate the following functions with respect to x : $10^x \csc x\ [\log 10 - \cot x]$

EXERCISE-2

1.   If $x < 2$, then write the value of $\dfrac{d}{dx} \left(\sqrt{x^2 - 4x + 4} \right)$.

2.   If $f(x) = \dfrac{x^2}{|x|}$, write $\dfrac{d}{dx}(f(x))$.

3.   If $|x| < 1$ and $y = 1 + x + x^2 + x^3 + ...$, then write the value of $\dfrac{dy}{dx}$.