MATHEMATICS

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MATHEMATICS
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DETERMINANTS

& Their Properties

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THINGS TO REMEMBER

✷ Determinant

An expression which is related to a squae matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n_1} & a_{n_2} & \dots & a_{nn} \end{bmatrix}$$

is Known as a determinant of A . It is denoted by det(A), or | A | and written as

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n_1} & a_{n_2} & \dots & a_{nn} \end{vmatrix}$$

Every square matrix can be associated to an expression or a number which is known as its determinant.

Value of a Determinant of Order 1

If $A = [a_{11}]$ is a square matrix or order 1 x 1.

then $|A| = a_{11}$

Value of a Determinant of Order 2

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a square matrix of order 2 x 2, then corresponding determinants is $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$= a_{11} a_{22} - a_{12} a_{21}$

Value of a Determinant of Order 3

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3 x 3 whose corresponding determinant is

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})s$$

eg, If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= (2 - 1) - 2(3 - 1) + 3(3 - 2)$$

$$= 1 - 2(2) + 3 = 4 - 4 = 0$$

✷ **Minors and Cofactors of a Determinant**

1. Minors

Let $A = [a_{ij}]$ is a square matrix of ordern, then minor of a_{ij} is a determinant of a square matrix of orer (n – 1) which is M_{ij}. M_{ij} is obtained on leaving *i*th row *j*th column of A.

$\therefore$ Minors of element a_1, a_2, a_3 of determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ are $\begin{vmatrix} b_2 & c_2 \\ b_3 & b_3 \end{vmatrix}$, $\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}$ and $\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ respectivey.

eg, If $A = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \\ 2 & -4 & 3 \end{vmatrix}$, the n

$$M_{11} = \begin{vmatrix} 2 & -1 \\ -4 & 3 \end{vmatrix} = 6 - 4 = 2$$

$$M_{12} = \begin{vmatrix} -3 & -1 \\ 2 & 3 \end{vmatrix} = -9 + 2 = -7$$

$$M_{13} = \begin{vmatrix} -3 & 2 \\ 2 & -4 \end{vmatrix} = 12 - 4 = 8$$

$$M_{21} = \begin{vmatrix} 2 & 3 \\ -4 & 3 \end{vmatrix} = 6 + 12 = 18$$

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 3 - 6 = -3$$

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 2 & -4 \end{vmatrix} = -4 - 4 = -8$$

$$M_{31} = \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} = -2 - 6 = -8$$

$$M_{32} = \begin{vmatrix} 1 & 3 \\ -3 & -1 \end{vmatrix} = -1 + 9 = 8$$

and $$M_{33} = \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} = 2 + 6 = 8$$

2. Cofactors

The cofactor C_{ij} of aij in Δ is equal to $(-1)^{i+j}$ times the determinant of order (n – 1) obtained by leaving *i*th row and *j*th column of Δ.

It follows that

Cofactor of an element a_{ij} of determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \text{ is } C_{ij}.$$

$\Rightarrow$ $\quad Cij = (-1)^{i+j} M_{ij}$. where M_{ij} is a minor of a_{ij}.

Cofactor of $a_{12} = C_{12} = (-1)^{1+2} M_{12}$.

$$= - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

eg, If $A = \begin{vmatrix} 4 & -7 \\ -3 & 2 \end{vmatrix}$, then cofactor are

$$C_{11} = (-1)^{1+1} M_{11} = M_{11} = 2$$

$$C_{12} = (-1)^{1+2} M_{12} = -M_{12} = -(-3) = 3$$

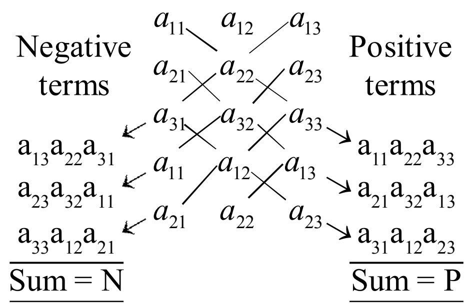
$$C_{21} = (-1)^{2+1} M_{21} = -M_{21} = 7$$

and $$C_{22} = (-1)^{2+2} M_{22} = M_{11} = 4$$

✱ **<u>Sarrus Rule</u>**

Write down the three rows of the determinant and rewrite the first two rows. the three diagonals sloping down to the right given the three positive terms and the three diagonals sloping down to the left given the three negative terms.

If $$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$



$\therefore \qquad \Delta = P - N$

✷ **Properties of a Determinant**

1. The value of the determinant remains uncharged, if rows are changed into columns and columns are changed into rows, ie,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

eg,
$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

2. If two adjacent rows (or columns) of a determinant are interchanged, the value of the determinant, so obtained is the negative of the value of the original determinant, ie,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

eg,
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix}$$

3. If two rows (or columns) of a determinant are identical, then its value is zero, ie,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

eg,
$$\begin{vmatrix} a & a & c \\ b & b & a \\ c & c & b \end{vmatrix} = 0$$

Here, first and second columns are same.

4. Each element of a row (or column) of a determinant is multiplied by a constant k, then the value of the new determinant is k times the value of the original determinant, ie,

$$\begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

eg, $$\begin{vmatrix} 2 & 4 & 6 \\ 4 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

5. If any two rows (or columns) of a determinants are proportional, then its value is zero, ie,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

eg, $$\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

6. If each element of a row (or column) of a determinant is the sum of two or more terms, then the detrminant can be expressed as the sum of the two or more determinants, ie,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+c_1 & a_{22}+c_2 & a_{23}+c_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ c_1 & c_2 & c_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

eg, $$\begin{vmatrix} a & b & c \\ d+e & f+g & h+i \\ j & k & l \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & f & h \\ j & k & l \end{vmatrix} + \begin{vmatrix} a & b & c \\ e & g & i \\ j & k & l \end{vmatrix}$$

7. If each element of a row (or column) of a determinant is multiplied by a constant k an then added to (or subtracted) from the corresponding elements of any other row (or column), then the value of the determinant remains the same, ie,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+ka_{21} & a_{31}+ka_{22} & a_{33}+ka_{23} \end{vmatrix}$$

eg, $$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g+ak & h+bk & i+ck \end{vmatrix}$$

8. If each element of a row (or column) of a determinant is zero, then its value is zero.
9. If r rows (or r column) become identical when a is substitued for x, then $(x - a)^{r-1}$ is a factor of given determinat.
10. The sum of the products of elements of any row (or column) of a determinant with the cofactors of the correspoinding elements of any other row (or column) is zero, ie, if

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \text{ then}$$

$a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33} = 0$ and so on.

11. The sum of the products of elements of any row (or column) of a determinant (where $|A|_{n \times n} = \Delta$) with the cofactors of the corresponding elements of same row (or column) is equal to Δ.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \text{ then}$$

$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = \Delta$.

Some Standard Results

1. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

2. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

3. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

4. $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

✶ **<u>Product of Determinants</u>**

Let Two determinants are

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$$

$$\text{Then, } \Delta_1 \cdot \Delta_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_3 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_3 + c_3\gamma_3 \end{vmatrix}$$

✷ Differentiation of Determinant

1. Let $\Delta(x) = \begin{vmatrix} a_{11}(x) & a_{12}(x) & a_{13}(x) \\ a_{21}(x) & a_{22}(x) & a_{23}(x) \\ a_{31}(x) & a_{32}(x) & a_{33}(x) \end{vmatrix}$

   Where $a_{ij}(x)$ is a differentiable function, then

$$\frac{d}{dx}\Delta(x) = \begin{vmatrix} a'_{11}(x) & a'_{12}(x) & a'_{13}(x) \\ a_{21}(x) & a_{22}(x) & a_{23}(x) \\ a_{31}(x) & a_{32}(x) & a_{33}(x) \end{vmatrix} + \begin{vmatrix} a_{11}(x) & a_{12}(x) & a_{13}(x) \\ a'_{21}(x) & a'_{22}(x) & a'_{23}(x) \\ a_{31}(x) & a_{32}(x) & a_{33}(x) \end{vmatrix} + \begin{vmatrix} a_{11}(x) & a_{12}(x) & a_{13}(x) \\ a_{21}(x) & a_{22}(x) & a_{23}(x) \\ a'_{31}(x) & a'_{32}(x) & a'_{33}(x) \end{vmatrix}$$

2. Let $\Delta(x) = \begin{vmatrix} a_{11}(x) & a_{12}(x) & a_{13}(x) \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

   Where $a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$ are constants, then

$$\frac{d}{dx}\Delta(x) = \begin{vmatrix} a'_{11}(x) & a'_{12}(x) & a'_{13}(x) \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

✷ Integration of Determinant

Let $\Delta(x) = \begin{vmatrix} a_{11}(x) & a_{12}(x) & a_{13}(x) \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\int \Delta(x)dx = \begin{vmatrix} \int a_{11}(x)dx & \int a_{12}(x)dx & \int a_{13}(x)dx \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

✷ Applications of Determinant in Geometry

1. Area of Triangle

If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

eg, the area of the triangle with vertices A(5, 4), B(–2, 4) and C(2, –6)

$$= \frac{1}{2}\begin{vmatrix} 5 & 4 & 1 \\ -2 & 4 & 1 \\ 2 & -6 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 5 & 4 & 1 \\ -7 & 0 & 0 \\ -3 & -10 & 0 \end{vmatrix} \qquad \begin{pmatrix} R_2 \to R_2 - R_1 \\ and\ R_3 \to R_3 - R_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -7 & 0 \\ -3 & -10 \end{vmatrix}$$

$$= \frac{1}{2}(70) = 35 \text{ sq unit}$$

2. Condition of Collinearty of Three Points

Let three points are A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3), then these points will be collinear, if

Area of ΔABC

$$\Rightarrow \qquad \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

3. Equation of Straight Line Passing Through Two Points

Let two points are A(x_1, y_1) and B(x_2, y_2) and P(x, y), be a point on the line joining points A and B, then equation of line is given by

$$\Rightarrow \qquad \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

✱ **Solution of System of Linear Equations**

Let $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

A set of values of variables x, y, z which simultaneously satisfy these three equations is called a solution.

Consistent

If the system of equations has a unique solution of infinite many solution, then the system of equations is known as consistent.

Inconsistent

If the system of equaations has no solution, then system of equations is known as inconsistent.

Trivial and Non-trivial Solution

If the value of all variables of system of equations is zero, ie, x = 0, y = 0 and z = 0, then solution is known as trivial solution.

If system of equations has infinite many solution, then the solution is known as non-rtrivial solution.

Homogeneous and Non-homogeneous System

If $d_1 = d_2 = d_3 = 0$ in given system of equations, then system of equations is said to be homogeneous otherwise it is said to be non-homogeneous system.

Cramer's Rule

A system of simultaneous linear equations can be solved by Cramer's rule named for the Swiss mathematician Gabriel Cramer.

1. The solution of the system of linear equations

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

is given by $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ where,

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \text{ and } D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

Provided that $D \neq 0$

2. Let the determinant of coefficient of equtions.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

and $a_3x + b_3y + c_3z = d_3$ is D.

$$\therefore \quad D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & c_3 & c_3 \end{vmatrix}$$

If $D \neq 0$, then

where, $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$; $D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$;

and $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Conditions for Consistency

1. If $D \neq 0$, then system of equation is consistent and has a unique solution given by

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad \text{and} \quad z = \frac{D_3}{D}$$

2. If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then system of equation is consistent with infinitely many solution.
3. If $D = 0$ and atleast one of D_1, D_2, D_3 is non-zero, then system of equation is inconsistent.

Solution of Homogeneous System of Equations

Let
$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

be a homogeneous system of linear equations. Then,

1. System of equations has a unique trivial solution

$$x = y = z = 0, \text{ if } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

2. System of equations has non-trivial solution, if $D = 0$.

Note :

- An arrangement of numbers is known as matrix and a matrix having equl number of rows and columns in known as square matrix.
- Matrix has no definite value while each diterminant has a difinite value.
- Determinant of a martix which is not a square matrix, cannot be find out.
- The determinant of order greater then 2 can be expanded along any row or column.
- This method is not application for determinants of order greater then 3.
- In $\Delta = |a_{ij}|$ is a determinant of order n, then the value of the determinant $|A_{ij}|$, where A_{ij} is the cofactor of a_{ij}, is Δ^{n-1}.
- If A = B + C, then ti is not necessary that

  $$\det(A) = \det(B) + \det(C)$$
- If A is a square matrix of orer n x n, then

  $$\det(kA) = kn(\det A)$$
- If A, B, C are three square matrix such that ith row of A is equal to the sum of ith row of B and C and remaining rows of A, B and C are same, then

  $$\det(A) = \det(B) + \det(C)$$
- $\det(A^n) = \det(A)^n$, where n is a positive integer.
- $\det(A^{-1}) = \dfrac{1}{\det(A)}$
- This formula is applicable only, if their is variable only in one row or column, otherwise expand the determinant and then integrate.