MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

DIFFERENTIABILITY

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. A real valued function f(x) defined on (a, b) is said to be differentiable at $x = c \in (a, b)$, iff

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} \text{ exists finitely.}$$

$$\Leftrightarrow \quad \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

$$\Leftrightarrow \quad \lim_{x \to 0} \frac{f(c - h) - f(c)}{-h} = \lim_{x \to 0} \frac{f(c - h) - f(c)}{h}$$

$\Leftrightarrow$ (LHD at x = c) = (RHD at x = c)

2. A function is said to be differentiable, if it is differentiable at every point in its donation.
3. Every differentiable function is continuous but, the converse is not necessarily true.
4. Following are some results on differentiability :
   (i) Every polynomial function is differentiable at each $x \in R$.
   (ii) The exponential function a^x, $a > 0$ is differentiable at each $x \in R$.
   (iii) Every constant function is differentiable at each $x \in R$.
   (iv) The logarithmic function is differentiable at each point in its domain.
   (v) Trigonometric and inverse–trigonometric functions are differentiable in their respective domains.
   (vi) The sum, difference, product and quotient of two differentiable functions is differentiable.
   (vii) The composition of differentiable function is a differentiable function.
   If a function f(x) is differentiable at every point in its domain, then

$$\lim_{x \to 0} \frac{f(x - h) - f(x)}{h} \quad \text{or,} \quad \lim_{x \to 0} \frac{f(x - h) - f(x)}{-h}$$

is called the derivative or differentiation of f and x and is denoted by f′(x) or $\frac{d}{dx} f(x)$.

EXERCISE-1

1. Show that $f(x) = |x|$ is not differentiable at x = 0.
2. Show that the function $f(x) \begin{cases} x - 1 & \text{if } x < 2 \\ 2x - 3 & \text{if } x \ge 2 \end{cases}$ is not differentiable at x = 2.
3. Show that the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & ,\text{if } x \ne 0 \\ 0 & ,\text{if } x = 0 \end{cases}$ is differential at x = 0 and f′(0) = 0.
4. Show that $f(x) = x^2$ is differentiable at x = 1 and find f′(1).
5. Discuss the differentiability of

$$f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} xe^{-2/x} & , x \neq 0 \\ 0 & , x = 0 \end{cases} \text{ at } x = 0.$$

6. If f(x) is differentiable at x = a, find $\lim_{x \to a} \frac{xf(2) - 2f(x)}{x - 2}$

7. Discuss the differentiability of f(x) = x| x | at x = 0.

8. Show that the function $f(x) = \begin{cases} x \sin\frac{1}{x} & , \text{where } x \neq 0 \\ 0 & , \text{where } x = 0 \end{cases}$ is continuous but not differentiable at x = 0.

9. Show that $f(x) = x^{1/3}$ is not differentiable at x = 0.

10. Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a & , \text{if } x \leq 1 \\ bx + 2 & , \text{if } x > 1 \end{cases}$ is differentiable each x ∈ R.

11. Show that the function f defined as follows, is continuous at x = 2, but not differentiable thereat :

$$f(x) = \begin{cases} 3x - 2 & , 0 < x \leq 1 \\ 2x^2 - x & , 1 < x \leq 2 \\ 5x - 4 & , x > 2 \end{cases}$$