MATHEMATICS

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AIM POINT
MATHEMATICS
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INEQUALITIES

& Their Properties

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THINGS TO REMEMBER

Inequality

An equation having signs >, <, > or < at the place of sign of equality (=) is known as inequality or inequation.

Properties of Inequalities

(i) If $a > b$ and $b < c$, then $a > c$. Generally, if $a_1 > a_2$, $a_2 > a_3$,..................., $a_2 > a_3$, thn $a_1 > a_n$.

(ii) If $a > b$, then $a \pm c > b \pm c$, $\forall\ c \in R$.

(iii) If $a > b$, then

for $m > 0$, $am > bm$, $\frac{a}{m} > \frac{b}{m}$.

and for $m < 0$, $bm > am$, $\frac{b}{m} > \frac{a}{m}$.

(iv) If $a > b > 0$, then

(a) $a^2 > b^2$ (b) $|a| > |b|$ (c) $\frac{1}{a} < \frac{1}{b}$.

and if $a < b < 0$, then

(a) $a^2 > b^2$ (b) $|a| > |b|$ (c) $\frac{1}{a} > \frac{1}{b}$.

(v) if $a < 0 < b$, then

(a) $a^2 > b^2$, if $|a| > |b|$

(b) $a^2 < b^2$, if $|a| < |b|$

(vi) If $a < x < b$ and a, b are positive real number, then

$$a^2 < x^2 < b^2$$

(vii) If $a < x < b$ and a is negative number and b is positive number, then

(a) $0 < x^2 < b^2$, if $|b| > |a|$

(b) $0 < x^2 < a^2$, if $|a| > |b|$

(viii) If $\frac{a}{b} > 0$, then

(a) $a > 0$, if $b > 0$ (b) $a < 0$, if $b < 0$

(ix) If $a_i < b_i < 0$, where $i = 1,2,3,..................,n$, then

$$a_1a_2a_3........a_n > b_1b_2b_3...........b_n.$$

(x) If $|x| < a$, then

(a) If a is positive, then $-a < x < a$

(b) If a is negative, then $x \in \phi$.

(xi) If $a_i < b_i$, where $i = 1,2,3,................,n$, then

$$a_1 + a_2 + a_3 + ... + a_n > b_1 + b_2 + ... + b_n.$$

(xii) If a > b > 0, then $\frac{1}{a} < \frac{1}{b}$

(xiii) If 0 < a < 1 and n is a prositive rational number, then

(a) $0 < a^n < 1$ (b) $a^{-n} > 1$

✶ Arithmetico-Geometric Mean Inequality

1. If a, b > 0 and a = b, then

$$\frac{a+b}{2} > \sqrt{ab}\,\frac{2}{\frac{1}{a}+\frac{1}{b}}$$

2. If $a_i < 0$, where i = 1,2,3,................,n, then

$$\frac{a_1+a_2+.....+a_n}{n} \geq (a_1 \cdot a_2 \cdot ..... \cdot a_n)^{1/n} \geq \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+.....+\frac{1}{a_n}}$$

Properties of Arithmetico-Geometric Mean Inequality

(i) If $a_1 = a_2 = ...= a_n$, then AM = GM

(ii) If a, b > 0 and a = b, then

(a) $\frac{a^m+b^m}{2} > \left(\frac{a+b}{2}\right)$, if m < 0 or m > 1

(b) $\frac{a^m+b^m}{2} < \left(\frac{a+b}{2}\right)$, if 0 < m < 1

(iii) If $a_i > 0$, where i = 1,2,3,...........,n, then

(a) $\frac{a_1^m+a_2^m+......+a_n^m}{n} > \left(\frac{a_1+a_2+.......+a_n}{n}\right)^m$, if m < 0 or m > 1

(b) $\frac{a_1^m+a_2^m+......+a_n^m}{n} < \left(\frac{a_1+a_2+.......+a_n}{n}\right)^m$, if 0 < m < 1

(iv) If a > 1 and n is any positive rational number, then

(a) $a^n > 1$ (b) $0 < a^{-n} < 1$

(v) If 0 < a < 1 and m, n are positive rational number, then

(a) $m > n = a^m < a^n$ (b) $m < n = a^m > a^n$.

(vi) (a) If a > 1 and $x > y > 0$, then $\log_a x > \log_a y$.

(b) If 0 < a < 1 and x > y > 0, then loga x < loga y

(vii) If a and b are positive real number, then

(a) $a < b \Rightarrow \frac{a+b}{b+c} > \frac{a}{b}; \forall c > 0$

(b) $a > b \Rightarrow \frac{a+b}{b+c} < \frac{a}{b}; \forall c > 0$

(viii) If a is a positive real number, then a + $\frac{1}{a} \geq 2$

and if a is a negative real number, then a + $\frac{1}{a} \leq -2$.

(ix) $|a + b| < |a| + |b|$

Generally, $|a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n| < |a_1| + |a_2| + |a_3| + \ldots\ldots\ldots + |a_n|$.

✷ Weighted Mean Inequality

1. If $a_i < 0$, where i = 1,2,3,..........,n and $m_1, m_2, \ldots\ldots, m_n$ are positive rational numbers, then

$$\left(\frac{m_1a_1 + m_2a_2 + \ldots\ldots + m_na_n}{m_1 + m_2 + \ldots\ldots + m_n}\right) \geq \left(a_1^{m_1} a_2^{m_2} \ldots\ldots a_n^{m_n}\right)^{1/(m_1+m_2+\ldots\ldots+m_n)}$$

and equality holds, iff

$a_1 = a_2 = \ldots\ldots\ldots = a_n$.

2. If $a_1, a_2, \ldots\ldots\ldots, a_n$ and $b_1, b_2, \ldots\ldots\ldots, b_n$ are rational numbers and m is a rational number, then

(a) $$\frac{b_1a_1^m + b_2a_2^m + \ldots\ldots + b_na_n^m}{b_1 + b_2 + \ldots\ldots + b_n} > \left(\frac{b_1a_1 + b_2a_2 + \ldots\ldots + b_na_n}{b_1 + b_2 + \ldots\ldots + b_n}\right)^m$$

If m < 0 or m < 1.

(b) $$\frac{b_1a_1^m + b_2a_2^m + \ldots\ldots + b_na_n^m}{b_1 + b_2 + \ldots\ldots + b_n} < \left(\frac{b_1a_1 + b_2a_2 + \ldots\ldots + b_na_n}{b_1 + b_2 + \ldots\ldots + b_n}\right)^m$$

If 0 < m < 1