MATHEMATICS

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MATHEMATICS
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MATRIX

& Their Properties

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THINGS TO REMEMBER

❋ Matrix

A rectangular array of mn numbers in the form of m horizontal line (called rows) and n vertical lines (called columns), is called a matrix of order m x n.

This type of array is enclosed by [ ] or () or || ||.

Each of mn numbers of a matrix is known as element of a matrix. A matrix is generally denoted by A, B, C,..........etc and its element is denoted by a_{ij} where a_{ij} belongs to the ith row and *j*th column and is called (i, j)th element of the matrix A = $[a_{ij}]$.

An m x n matrix is usully written as $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$

eg, $A = \begin{bmatrix} 3 & 2 & 7 \\ 5 & -4 & 6 \\ 4 & 8 & -12 \end{bmatrix}$ is a matrix of order 3 x 3.

❋ Types to Marices

1. Row Matrix

A matrix which has only one row and any mumber of colmns, is called a row matrix.

eg, $A = [27 \quad 85 \quad 1 \quad 4]_{1 \times 4}$ is a row matrix.

2. Column Matrix

A marix is said to be a column matrix, if it has only one column and any number of rows.

eg, $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1}$ is a column matrix.

3. Square Marix

A matrix in which number of rows is equal to the number of columns, is called a square matrix. The elements a_{ij} of a square matrix $A = [a_{ij}]_{m \times n}$ for which $i = j$ ie, the elements $a_{11}, a_{12}, \ldots\ldots, a_{mm}$, are called the diagonla elements and the line along which they lie is called the principal diagonal or leading diagonal of the matrix.

eg, $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}_{3 \times 3}$ is a square matrix of order 3 in which diagonal elements are 1, 2, 1.

4. Null Matrix

A matrix of order m x n whose all elements are zero, is called a null matrix of order m x n.

It is denoted by O.

eg, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are two null matrices of order 2 x 2 and 2 x 3 respectively.

5. Diagonal Matrix

A squae matrix is called a diaonal matrix, if all its non-diagonal elements are zero diagonal elements are not all equal.

If $d_1, d_2, d_3, ..., d_n$ are element of principal diagonal of a diagonal matrix of order n x n, then matrix is denoted as

diag $[d_1, d_2, ..., d_n]$.

eg, $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ is a diagonal matrix, which is denoted by

A = diag [a, b, c]

6. Scalar Matrix

A square matrix $A = [a_{ij}]$ is said to scalar matrix, if

(a) $a_{ij} = 0, \ \forall \ i \neq j$

(b) $a_{ij} = k, \ \forall \ i = j$, where $k \neq 0$

ie, a diagonal matrix is said to be a scalar matix, if the elements of principal diaonal are same.

eg, $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix.

7. Unit Matrix

A square matrix A = [aij] is said to be a unit matrix or identity matrix, if

(a) $a_{ij} = 0, \ \forall \ i \neq j$

(b) $a_{ij} = 1, \ \forall \ i = j$

ie, a diagonal matrix, whose elements of principal diagonal are equal to 1 and all remaining elements are zero, is known as unit or identity matrix. It is denoted by I.

eg, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a unit matrix or order 3.

8. Upper Triangular Matrix

A square square matrix A = $[a_{ij}]$ is known a upper triangular matrix, If $a_{ij} = 0, \ \forall \ i > j$.

eg, $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$ is an upper triangular matrix.

9. Lower Triangular Matrix

A square square matrix A = $[a_{ij}]$ is known a upper triangular matrix, If $a_{ij} = 0, \ \forall \ i < j$.

eg, $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$ is an lower triangular matrix.

✵ **Equality of Two Matrices**

Two matrice $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are said to be equal, if

1. number of rows in A is equal to number of rows in B.
2. number of column in A is equal to number of columns in B.
3. $A_{ij} = b_{ij}, \ \forall \ i = 1, 2, ..., m$ and $j = 1, 2, ...,n$

✵ **Trace of a Martix**

Let $A = [a_{ij}]_{m \times n}$ be a square matrix. Then, the sum of all diagonal elements of A is called the trace of A and is denoted by tr(A).

Thus, $$\text{tr}(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + ... + a_{nn}$$

eg, $$A = \begin{bmatrix} 2 & -7 & 9 \\ 0 & 3 & 2 \\ 8 & 9 & 4 \end{bmatrix}$$

$$\text{tr}(A) = 2 + 3 + 4 = 9$$

Properties of Trace of a Matrix

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ are two square matrix of order n, then

(i) tr (A + B) = tr(A) + tr(B)

(ii) tr (AB) = tr (BA)

(iii) tr (λA) = λtr(A), where λ is a scalar.

✵ **Algebra of Matrices**

Let $A = [a_{ij}]_{m \times n}$ and B = [bij]m x n are two matrices whose orders are same, then.

$A + B = [a_{ij} + b_{ij}]$, i = 1, 2,......, m and j = 1, 2, ..., n

Also, A – B = A + (–B)

eg, If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

Then, $$A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

Properties of Matrix Addition

Let A, B and C are three matirces of same order, then

(a) Matrix addition is coommutive, ie,

$$A + B = B + A$$

(b) Matrix addition is associative, ie,

$$(A + B) + C = A + (B + C)$$

(c) If O is a null matrix of order m x n and A + O = O + A, then O is known as additive identity.

(d) If for each matrix $A = [a_{ij}]_{m \times n}$ a matrix (–A) is such that

$$A + (-A) = O = (-A) + A,$$

then matrix (–A) is known as additive inverse of A.

(e) Matrix addition follows cancellation law, ie,

$A + B = A + C \Rightarrow B = C$ (Left cancellation law)

and $B + A = C + A \Rightarrow B = C$ (Rigt cancellation law)

Saclar Multiplication

Let $A = [a_{ij}]$ be any mx n marix and k be any scalar. Then, the matrix obtained by multiplying each element of A by k is called the scalar multiple of A by k and is denoted by kA.

Thus, If $A = [a_{ij}]_{m \times m}$ then $kA = [ka_{ij}]_{m \times n}$.

eg, If $$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

then $$2A = \begin{bmatrix} 2 & 4 & 6 \\ 6 & 4 & 2 \\ 2 & 6 & 2 \end{bmatrix}$$

Proerties of scalar Multiplication If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices and λ and μ are two scalars, then

1. $\lambda(A + B) = \lambda A + \lambda B$
2. $(\lambda + \mu)A = \lambda A + \mu B$
3. $(\lambda)\mu A = \mu(\lambda A) = \lambda(\mu A)$
4. $(-\lambda)A = -(\lambda A) = \lambda(-A)$

Multiplication of Two Matrices

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ are two matrices such that the number of column of A is equal t the number of rows of B, Then a matrix $C = [c_{ij}]_{m \times p}$ of order m x p is known as product of matrices A and B, where

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1} b_{1j} + a_{i2} b_{2j} + ... + a_{in} b_{nj}.$$

Multiplication of matrices is denoted by C = AB.

eg, If $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 1 & 7 \end{bmatrix}$, then

$$AB = \begin{bmatrix} 2\times 5+1\times 1 & 2\times 2+1\times 7 \\ 3\times 5+5\times 1 & 3\times 2+5\times 7 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 \\ 20 & 41 \end{bmatrix}$$

Properties of Multiplication of Matrices Let $A = [a_{ij}]_{m \times m}$ $B = [a_{ij}]_{n \times p}$ and $C = [c_{ij}]_{p \times m}$ are three matrices, then

(a) Generally, matrix multiplication is not commutative, ie,

$$AB \neq BA$$

(b) Matrix multiplication is associative, ie,

$$A(BC) = (AB)C$$

(c) Matrix multiplication is distributive over matrix addition.

$$A(B + C) = AB + AC$$

(d) If A is a m x n matrix and In is an identity matrix of order n x n and Im is an identity matrix of order m x m, then

$$I_m A = A \quad \text{and} \quad AI_n = A$$

In particular, if A is a square matrix of order n, then

(e) AB = O does not necessarily imply that A = O or B = O or both A and B are O.

eg, If $A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \neq O$

and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \neq O$

But $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

❋ Transpose of a Matrix

If $A = [a_{ij}]_{m \times n}$ is a matrix of order m x n, then the transpose of A can be obtained by changing all rows to columns and all columns to rows ie, transpose of $A = [a_{ij}]_{n \times m}$. It is denoted by A', A^T or A^t.

eg, If $A = \begin{bmatrix} 1 & 3 & 5 \\ 6 & 9 & 4 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & 6 \\ 3 & 9 \\ 5 & 4 \end{bmatrix}$

Properties of Transpose

If A and B are two matrices and k is a scalar, then

(i) $(A')' = A$

(ii) $(A + B)' = A' + B'$

(iii) $(kA)' = kA'$.

(iv) $(AB)' = B'A'$ (Reversal Law)

❋ Conjugate of a Matrix

The matrix obtained from any given matrix A containing complex numbers as its elements, on replacing its elements by the corresponding conjugate complex number is called conjugate of A and is denoted by $\overline{A}$.

eg, If $A = \begin{bmatrix} 1+2i & 2-3i \\ 4-5i & 5+6i \end{bmatrix}$,

then $\overline{A} = \begin{bmatrix} 1-2i & 2+3i \\ 4+5i & 5+6i \end{bmatrix}$

Properties of Conjugate

If A and B are two matrices and k is a scalar, then

(i) $\overline{(\overline{A})} = A$

(ii) $\overline{(A+B)} = \overline{A} + \overline{B}$

(iii) $\overline{(kA)} = k\,\overline{A}$

(iv) $\overline{AB} = \overline{A} \cdot \overline{B}$

❋ Conjugate Transpose of a Matrix

The transpose of the conjugate of a matrix A is called conjugate transpose of A and is denoted by A^{Θ} or A^{*}.

$$A^{\Theta} = \text{Conjugate of } A' = \overline{(A')}$$

The transpose of the conjugate of A is the same as the conjugate of the transpose of A.

eg, If $$A = \begin{bmatrix} 2+4i & 3 & 5-9i \\ 4 & \alpha+\beta i & 3i \\ 2 & -5 & 4-i \end{bmatrix},$$

then $$A^{\Theta} = \overline{(A')} = \begin{bmatrix} 2-4i & 4 & 2 \\ 3 & \alpha-\beta i & -5 \\ 5+9i & -3i & 4+i \end{bmatrix}$$

Properties of Conjugate Transpose

(i) For a matrix A

$$\overline{(A')} = \left(\overline{A}\right)'$$

(ii) $(A^{\Theta})^{\Theta} = A$

(iii) If A and B are two matrices, then

$$(A + B)^{\Theta} = A^{\Theta} + B^{\Theta}$$

(iv) $(kA)^{\Theta} = kA^{\Theta}$, where k is any scalar.

(v) $(AB)^{\Theta} = B^{\Theta}A^{\Theta}$

❋ Special Types of Matrices

1. Nilpotent Matrix

If $A^k = O$ and $A^{k+1} \neq O$, where k is a positie integer and O is a null matrix, then A is called nilpotent matrix, k is called the index of the nilpotent matrix A.

2. Periodic Matrix

If $A^{k+1} = A$, where k is a positive integer, then A is known as periodic matirix, k is known as period of matrix A.

For $k = 1$, $A^2 = A$

3. Idempotent Matrix

If $A^2 = A$, then square matrix A is known as idempotent matrix.

4. Involutory Matrix

If $A^2 = I$, where I is an identity matrix, then A is called an involutory matrix.

5. Symmetric Matrix

If for a square matrix A, $A' = A$, then A is known as symmetric matrix.

eg, If $A = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$,

then $A' = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \Rightarrow A' = A$

6. Skew-symmetric martix

A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a skew-symmetric matrix, if

1. $a_{ij} = -a_{ji}, \ \forall$ i, j
2. Each element of diagonal is zero.

OR

A square matrix A is said to be a skew symmetric matrix, if $A' = -A$.

eg, if $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 0 \end{bmatrix}$,

then $A' = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -3 \\ -3 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & -3 \\ 3 & -3 & 0 \end{bmatrix} = -A$

Hence, A is a skew-symmetric matrix.

7. Orthogonal Matrix

If the product of a square matrix and its transpose A is an identity matrix, then matrix A is said to be an orthogonal matrix.

ie, $AA' = I = A'A$

eg, Let $A = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

$\therefore$ $A' = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$

Also, $AA' = \frac{1}{2\sqrt{2}} \cdot \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$

$= \frac{1}{8}\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \qquad AA' = I$

Similarly, $\qquad A'A = I$

Thus, A is an orthogonal matrix.

8. Hermitian Matrix

A square matrix A such that $A^{\Theta} = \overline{A}' = A$, then A is known as hermitian matrix.

eg, Let $\qquad A = \begin{bmatrix} \alpha & \lambda + i\mu \\ \lambda - i\mu & \beta \end{bmatrix}$

$\therefore \qquad A' = \begin{bmatrix} \alpha & \lambda - i\mu \\ \lambda + i\mu & \beta \end{bmatrix}$

$$A^{\Theta} = (\overline{A}') = \begin{bmatrix} \alpha & \lambda + i\mu \\ \lambda - i\mu & \beta \end{bmatrix} = A$$

$\therefore \qquad A^{\Theta} = A$

Thus, A is a hermitian matrix.

9. Skew-hermitian Matrix

A square matrix A such that $A^{\Theta} = -A$, then A is known as skew-hermitian matrix.

eg, Let $\qquad A = \begin{bmatrix} 2i & -2-3i \\ 2-3i & -i \end{bmatrix}$

$\therefore \qquad A' = \begin{bmatrix} 2i & -2-3i \\ 2-3i & -i \end{bmatrix}$

$$A^{\Theta} = (\overline{A}')$$

$$= \begin{bmatrix} -2i & 2+3i \\ -2+3i & i \end{bmatrix}$$

$$= -\begin{bmatrix} 2i & -2-3i \\ 2-3i & -i \end{bmatrix} = -A$$

$\therefore \qquad A^{\Theta} = -A$

Thus, A is a skew-hermitian matrix.

10. Unitary Matrix

If for a square matrix A, $AA^{\Theta} = I$, then A is known as unitary matrix.

11. Singular and Non-singular Matrix

If A is a square Matrix and $|A| = 0$, then A is known as singular matrix.

eg, Let $\qquad A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

and $|A| = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = 6 - 6 = 0$

$\therefore$ A is a singular matrix.

If A is a square matrix and $|A| = 0$, then A is known as non-singular matirx.

eg, Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$$|A| = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = 4 - 6 = -2 \neq 0$$

$\therefore$ A is a non-singular matrix.

✵ Adjoint of a square Matrix

Let $A = [a_{ij}]_{n \times n}$ is a square matrix order n and let C_{ij} be the cofactor of aij in the determinant $|A|$. Then, the adjoint of A is denoted by adj(A) and is defind as the transpose of the cofactor matrix.

Properties of Adjoint of a square Matrix

1. $(\text{adj } A)A = A(\text{adj } A) = |A| \cdot I_n$
2. $|\text{adj } A| = |A|^{n-1}$, if $|A| \neq 0$.
3. $\text{adj } (AB) = (\text{adj } B)(\text{adj } A)$
4. If $|A| = 0$, then $(\text{adj } A)A = A(\text{adj } A) = 0$
5. $\text{adj } (\text{adj } A) = |A|^{n-2} A$, where A, is a non-singular matrix.
6. $\text{adj } (A^T) = (\text{adj } A)^T$.
7. Adjoint of a diagonal matrix is a diagonal matrix.

✵ Inverse of a Martix

If two square marices of same order are A and B, for which

$$AB = BA = I_n$$

Then, B is known as inverse of A, ie,

$$A^{-1} = B$$

If $|A| = 0$ ie, A is non-singular, then $A^{-1} = \dfrac{adj\ A}{|A|}$.

Properties of Inverse of a Matrix

If A, B and C are three matrices of same order and $|A| \neq 0$, $|B| \neq 0$ and $|C| \neq 0$, then

1. (a) $AB = AC \Rightarrow B = C$ (Left Cancellation law)

   (b) $BA = CA \Rightarrow B = C$ (Right Cancellation law)
2. (a) $(AB)^{-1} = B^{-1}A^{-1}$

   (b) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
3. $(A^T)^{-1} = (A^{-1})^T$

4. $(kA)^{-1} = A^{-1}$, if $k \neq 0$

5. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $|A| \neq 0$, then

$$A^{-1} = \frac{1}{ad - bx}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

6. If A is a non-singular matrix, then

$$|A^{-1}| = |A|^{-1} \quad |A^{-1}| = \frac{1}{|A|}$$

7. If A is a symmetric matrix, then A^{-1} is also a symmetric matrix.

8. A square matrix is invertible iff it is non-singular and every invertiable matrix possesses a unique inverse.

✵ Elementary Row Transformations

Any one of the following operations on a matrix is called an elementary row (or column) transformation.

(a) Interchanging any two rows (or columns). This transformation is indicated by

$$R_1 \leftrightarrow R_j (or \ C_i \leftrightarrow C_j)$$

(b) Multiplication of the elements of any row (or column) by a non-zero scalar quantity. This transformation is indicated as

$$R_i \leftrightarrow kR_i \ (or \ C_i \leftrightarrow kC_i)$$

(c) Addition of a constant multiple of the elements of any row to the corresponding element of any other row. This transformation is indicated as $R_i \rightarrow R_i + kR_i$.

Elementary Matrix

A matrix obtained from an identity matrix by a single elementary operation is called an elementary matrix.

Inverse of a Matrix using Elementary Row Transformation

Let $\quad A = IA$

If matrix A (LHS) is reduced to I by elementary row transformation, then suppose I (RHS) is reduced to P and not change A in RHS is, after transformation we get, I = PA,

then P is the inverse of A,

$\therefore \quad P = A^{-1}$

✵ Solution of Simultaneous Linear Equations

Let n simulatneous linear equation in n unknowns are

$$a_{11}x_1 + a_{12}x_2 + \dots\dots\dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots\dots\dots + a_{2n}x_n = b_2$$
$$\vdots \quad \vdots \qquad \vdots \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots\dots\dots + a_{nn}x_n = b_n,$$

then system of equations can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1}$$

or $$AX = B$$

Where, matrix A is called coefficient matrix, matrix X is called variable matrix and matrix B is called column matrix of given constants.

Solution of Non-homogeneous System of Equation

If $B \neq 0$, then given system of equation of equations $AX = B$ is non-homogeneous.

(i) If $|A| \neq 0$, then the system of equations is consistent and has a unique solution given by $X = A^{-1}B$.

(ii) If $|A| = 0$ and $(\text{adj } A)\, B \neq O$, then the system of equations is inconsistent and has no solution.

(iii) If $|A| = 0$ and $(\text{adj } A).\, B = O$, then the system of equations is consistent and has an infinite number of solutions.

Solution of Homogeneous System of Equations

If $B = 0$, then given system of equation $AX = B$ is homogeneous.

(i) If $|A| = 0$, the system of equation has only trivial solution and it has one solution.

(ii) If $|A| = 0$, the system of equation has non-trivial solution and it has infinite solutions.

(iii) If number of equation < number of unknowns, then it has non-trivial solution.

Note :

- A matrix is only an arrangement of numbers, it has no definite value.
  eg, $[7] \neq 7$
- The elements of matrix may be scalar or vector quantity.
- Two matrices are said to be conformable for addition or subtraction iff they are of the same order.
- In multiplication of two matrices A and B, the order roles an important role.
- If A,B,C are any three matrices conformable for multiplication, then $(ABC)' = C'\,B'\,A'$
- If A and B are two orthogonal matrices then AB will also be an orthogonal matrix.
- Adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing the signs of the off-diagonal elements.
- Two matrices are said be equivalant, if one is obtained from the other by elementary tranformations. The symbol $\approx$ is used for equivalence.