MATHEMATICS

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AIM POINTS MATHEMATICS
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RELATIONS
& Their Properties

CONTENTS

THINGS TO REMEMBER

1. $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2$ and b_1 b_2
2. If A and B are two non–empty sets, then
   $A \times B = \{(a, b) : a \in A, b \in B\}$ is called cartesian product of A and B.
   If A and B are finite sets having m and n elements respectively, the $A \times B$ has mn elements.
3. $R \times R = \{(x, y) : x, y \in R\}$ is the set of all points in xy–plane.
4. For any three sets A, B, C, we have :
   (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
   (iii) $A \times (B - C) = A \times B - A \times C$
   (iv) $A \times B = B \times A = B$
   (v) $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$
   (vi) $A \times (B' \cup C')' = (A \times B) \cup (A \times C)$
   (vii) $A \times (B' \cap C')' = (A \times B) \cap (A \times C)$
   (viii) $A \times B = A \times C \Rightarrow B = C$
5. If R is a relation from set A to set B, then Domain (R) = $\{x : (x, y) \in R\}$, Range (R) = $\{y:(x,y)\in R\}$.

EXERCISE–1

1. Order Pair.
2. Equality of ordered pairs.
3. Cartesian Product of sets.
4. If $A = \{2, 4, 6\}$ and $B = \{1, 2\}$, then $A \times B$ and $B \times A = ?$
5. If $A = \{1, 2\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Then, $A \times B \times C = ?$
6. If A and B are two finite sets, then $n(A \times B) = n(A) \times n(B)$.
7. Graphical presentation of cartesian productof sets.
8. Diagramatic representation of cartesian product of two sets.
9. Find x and y, if $(x + 3, 5) = (6, 2x + y)$.
10. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{1, 3, 5\}$, find
    (i) $A \times (B \cup C)$
    (ii) $A \times (B \cap C)$
    (iii) $(A \times B) \cap (A \times C)$
11. Let $A = \{1, 2, 3\}$ and $B = \{x : x \in N, x$ is prime less than $5\}$. Find $A \times B$ and $B \times A$.
12. If R is the set of all real numbers, what do the cartesian products $R \times R$ and $R \times R \times R$ represent ?
13. Express $A = \{(a, b) : 2a + b = 5, a, b \in W\}$ as the set of ordered pairs.
14. Let A and B be two sets such that $A \times B$ consists of 6 elements. If three elements of $A \times B$ are : (1, 4), (2, 6), (3, 6). Find $A \times B$ and $B \times A$.
15. Let A and B be two sets such that $n(A) = 5$ and $n(B) = 2$. If a, b, c, d, e are distinct and (a, 2), (b, 3), (c, 2), (d, 3), (e, 2) are in $A \times B$, find A and B.
16. Let $A = \{-1, 3, 4\}$ and $B = \{2, 3\}$. Represent the following products graphically i.e. by lattices :
    (i) $A \times B$
    (ii) $B \times A$
    (iii) $A \times A$

17. If a $\in$ {–1, 2, 3, 4, 5} and b $\in$ {0, 3, 6}, write the set of all ordered pairs (a, b) such that a + b = 5.
18. If a$\in$ {2, 4, 6, 9} and b $\in$ {4, 6, 18, 27}, then form the set of all ordered pairs (a, b) such that a divides b and a < b.
19. If A = {1, 2, 3} and B = {2, 4}, what are A × B, B × A, A × A, B × A and (A × B) $\cap$ (B × A) = ?
20. Let A and B be two sets. Show that the sets A × B and B × A have an element in common iff the sets A and B have an element in common.
21. Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y, z are distinct elements.
13. State whether each of the following statements are true or false. If the statement is false, re–write the given statement correctly :
    (i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}
14. If A = {1, 2, 4} and B = {1, 2, 3}, represent following sets graphically.
    (i) A × B (ii) B × A (iii) A × A (iv) B × B
15. For any three sets A, B, C, prove that :
    (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
    (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
16. For any three sets A, B, C, prove that : $A \times (B - C) = (A \times B) \cap (A \times C)$
17. If A and B are any two non–empty sets, then prove that : $A \times B = B \times A \Leftrightarrow A = B$
18. If $A \subseteq B$, show that $A \times A \subseteq (A \times B) \cap (B \times A)$
19. If $A \subseteq B$, prove that $A \times C \subseteq B \times C$ for any set C.
20. If $A \subseteq B$ and $C \subseteq D$, prove that $A \times C \subseteq B \times D$.
21. For any sets A, B, C, D prove that : $(A \times B) \cap (C \times D) = (A \cap C) \cap (B \times D)$
22. For any three sets A, B, C prove that
    (i) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
    (ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$
23. Let A and B two non–empty sets having n elements in common, then prove that A × B and B × A have n^2 elements in common.
24. Let A be a non–empty set such that A × B = A × C. Show that B = C.
25. Relations.
26. Representation of a relation.
27. Set–builder form.
28. By arrow diagram.
29. By lattice
30. Domain and range of a relation.
31. Inverse of a relation.
32. Let A = {1, 2, 3}, B = {a, b, c, d} be two sets and let R = {(1, a), (1, c), (2, d), 2, c)} be a relation from A to B. Then R^{-1} = {(1, a), (c,1), (d, 2), (c, 2)} is a relation from B to A.
33. If A = {1, 2,3}, B = {4, 5, 6}, which of the following are relations from A to B ? Give reasons in support of your answer.

(i) R_1 = {(1, 4), (1, 5), (1, 6)}
(ii) R_2 = {(1, 5), (2, 4), (3, 6)}
(iii) R_3 = {(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)}
(iv) R_4 = {(4, 2)] (2, 6), (5, 1), (2, 4)}

34. A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows : $(x, y) \in R \Leftrightarrow$ x divides y. Express R as a set of ordered pairs and determine the domain and range of R. Also find R^{-1}.

35. If R is the relation "less than" from A = {1, 2, 3, 4, 5} to B = {1, 4, 5}, write down the set of ordered pairs corresponding to R. Find the inverse of R.

36. Let R be the relation on the set N of natural numbers defined by R = (a, b) : a + 3b = 12, $a \in N$, $b \in N$}. Find :
(i) R
(ii) Domain of R
(iii) Range of R

37. The adjacent figure show R between the sets P and Q. Write this relation R in
(i) Set builder form
(ii) Roster form
What is the domain and range ?

38. Let R be a relation on Q defined by R = {(a, b) : a, b ∈ Q and a – b ∈ Z}
Show that :
(1) (a, a) ∈ R for all a ∈ Q
(ii) (a, b) ∈ R ⇒ (b, a) ∈ R
(iii) (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R

39. Let R be a relation on N defined by R {(a, b) : a, b ∈ N and $a = b^2$}

40. Let a relation R_1 on the set R of all real numbers be defined as $(a, b) \in R_1 \Leftrightarrow 1 + ab > 0$ for all a, b ∈ R.

41. If A = {1, 2, 3}, B = {4, 5, 6}, which of the following are relations from A to B ? Give reasons in support of your answer.
(i) {(1, 6), (3, 4), (5, 2)}
(ii) {(1, 5), (2, 6), (3, 4), (3, 6)}
(iii) {(4, 2), (4, 3), (5, 2)}
(iv) A × B

42. Let A be the set of first five natural numbers and let R be a relation on A defined as follows :
$(x, y) \in R \Leftrightarrow x \leq y$

43. Write the following relations as the sets of ordered pairs :
(i) A relation R from the set {2, 3, 4, 5, 6} to the set {1, 2,3} defined by x = 2y.
(ii) A relation R on the set {1, 2, 3, 4, 5, 6, 7} defined by $(x, y) \in R \Leftrightarrow$ x is relatively.

44. Determine the domain and range of the following relations :
(i) R = {(a, b) : a ∈ N, a < 5, b = 4}
(ii) S = {(a, b) : b | a – 1 |, a ∈ Z and | a | ≤ | 3 |

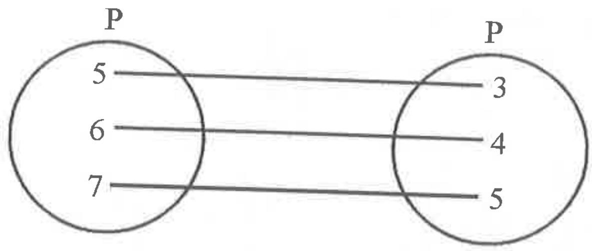
45. Let A = {1, 2, 3, ..., 14}. Define a relation on a set A by R = {(x, y) : 3x – y = 0, where x, y ∈ A}.

46. Define a relation R on the set N of natural numbers by R = {(x, y) : y = x + 5, x is a natural number less than 4, x, y ∈ N}. Depict this relationship using (i) roster form (ii) an arrow diagram. Write down the domain and range or R.

47. Write the relation $R = \{(x, x^3\}$: x is a prime number less than 10} in roster form.

48. Let A = {1, 2, 3, 4, 5, 6}. Let R be a relation on A defined by {(a, b) : a, b ∈ A, b is exactly divisible by a}.
   (i) Write R in roster form
   (ii) Find the domain of R
   (iii)Find the range of R.

49. The adjacent figure shows a relationship between the sets P and Q. Write this relation in (i) set builder form (ii) orster form. What is its domain and range ?



50. Let R be the relation on Z defined by R = {(a, b) : a, b ∈ Z, a – b is an integral.}

51. Let R be a relation on N × N defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ N ×B
   (i) (a, b) R (a, b) for a,, (a, b) ∈ N × N
   (ii) (a, b) R (c, d) ⇒ (c, d) R (a, b) for all (a, b), (c, d) ∈ N × N
   (iii)(a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f) for all (a, b), (c, d), (e, f) ∈ N × N

52. If R = {(x, y) : x, y ∈ Z, $x^2 + y^2 \le 4$} is a relation defined on the set Z of integers, then write domain of R.

53. Let A = {1, 2, 3} and R = {(a, b) : | $a^2 - b^2$ | ≤ 5, a, b ∈ A}. Then write R as set of ordered pairs.

54. If R = {(1, 2), (4, 7), (1, –2)...}, then write the linear relation between the components of the ordered pairs of the relation R.

55. If R = {(x, y) : x, y ∈ W, 2x + y = 8}, then write the domain and range of R.

56. Let A = {1, 2, 3, 5}, B = {4, 6, 9} and R be a relation from A to B defined by R = {(x, y) : x – y is odd}. Write R in roster form.