MATHEMATICS

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MATHEMATICS
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SETS

& Their Properties

CONTENTS

THINGS TO REMEMBER

✱ Sets

The German Mathematician "George Canter (1845-1918)" developed the theory of sets.

In our mathematical language all living and non-living things in universe are known as objects.

The collecation of well defined distinct objects is known as a set.

Well defined means in a given set it must be possible to decide whether or not the obbject belongs to the set and by distinct means object should not be repeated.

The object in the set is called its member of element.

Generally, sets are denoted by A, B, C,.....and its elements are denoted by a, b, c,.............

Let A is a non-empty set. If x is an element of A, then we write '$x \in A$' and read as "x is an element of A" or "x belongs of A". If x is not an element of A, then we write '$x \notin A$' and read as "x is not an element of A" or "x does not belong to A".

Representation of Sets

We can use the following two methods to represent a set.

(i) Listing method

(ii) Set builder method

(i) Listing method

In this method, elements are listed and put within a braces { } and separated by commas. This method is also known as Tabular method or Roster method.

eg, A = set of all prime numbers less than 11

$= \{2, 3, 5, 7\}$

(ii) Set builder method

In this method, instead of listing all elements of a set, we list the property of properties satisfied by the elements of set and write it as.

$$A = \{x : P(x)\}$$

It is read as "A is the set of all elements x such that x has the property P(x)." the symbol ':' stands for such that,

eg, $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{x : x \in N \text{ and } x \leq 8\}$

This method is also known as Rule method or property method.

✱ Different Types of Sets

1. Empty Set

A set which has no element, is called an empty set.

It is also known as void set or null set. It is denoted by f or { }.

eg, A = set of all odd numbers divisible by 2

B = $\{x : x \in N \text{ and } 5 < x < 6\}$

Such sets which have at least one element, are called non-void set.

2. Singleton Sets

A set which have only one element, is called a singleton set.

eg, A = $\{x : x \in N \text{ and } 3 < x < 5\}$

B = $\{5\}$

3. Finite and Infinite Sets

A set in which the process of counting of elements surely comes to an end, is called a fininte set. In other words "A set having finite number of elements of elements is called a finite set". Otherwise it is called inifite set ie, if the process of counting of elements does not come to an end in a set, then set is called an infinite set.

eg, A = $\{x : x \in N; x < 5\}$

B = set of all points on a plane

In above two sets A and B, set A is finite while set B is infinite. Since, on a plane any number of points are possible.

4. Equivalent Sets

Two finite sets A and B are said to be equivalent if they have the same number of elements.

eg, If A = $\{1, 2, 3\}$ and B = $\{3, 7, 9\}$

Number of elements in A = 3

and Number of elements in A = 3

$\therefore$ A and B are equivalent sets.

5. Equal Sets

If A and B are two non-empty sets and each element of set A is an element of set B and each element of set B is an element of set A, then sets A and B are called equal sets.

Symbolically, if $x \in A \Rightarrow x \in B$

and $x \in B \Rightarrow x \in A$

eg, A = $\{1, 2, 3\}$ and B = $\{x : x \in N; x \leq 3\}$

Here each element of A is an element of B, also each element of B is an elements of A, then both sets are called equal sets.

6. Subset and Superset

Let A and B be two non-empty sets. if each element of set A is an element of set B, then set A is known as subset of set B. If set A is a subset of set B, then set B is called the superset of A.

Also, if A is a subset of B, then it is denoted as $A \subseteq B$ and read as "A is subset of B".

Thus, if $x \in A \Rightarrow x \in B$

then $A \subseteq B$

If $x \in A \Rightarrow x \notin B$

then $A \not\subseteq B$

and read as "A is not a subset of B".

eg, If A = {1, 2, 3} and B = {1, 2, 3, 4, 5}

Here each element of A is an element of B. Thus, A B ie, A is a subset of B and B is a superset of A.

7. Proper Subset

If each element of A is in set B but set B has at least one element which is not in A, then set A is known as proper subset of set B. If A is a proper subset of B, then it is written as '$A \subset B$' and read a "A is a proper subset of B".

eg, If N = {1, 2, 3, 4,...............}

and I = {..........., –3, –2, –1, 0, 1, 2, 3, ...}

then $N \subset I$

8. Comparability of Sets

Two sets A and B are said to be comparable if either $A \subset B$ or $B \subset A$ or A = B, otherwise, A and B are said to be incomparable.

eg, A = {1, 2, 3}, B = {1, 2, 4, 6} and C = {1, 2, 4}

Since $A \not\subset B$ or $B \not\subset A$ or $A \neq B$.

$\therefore$ A and B are incomparable.

But $C \subset B$

$\therefore$ B and C are comparable.

9. Universal Set

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set and it is denoted by S or U.

This set can be chosen arbitrarily for and discussion of given sets but after choosing it is fixed.

eg, A = {1, 2, 3}, B = {3, 4, 5} and C = {7, 8, 9}

$\therefore$ U = {1, 2, 3, 4, 5, 6, 7, 8, 9} is universal set for all there sets.

10. Power Set

Let A be a non-empty set, then collection of all possible subsets of set A is known as power set. It is denoted by P(A).

eg, A = {1, 2, 3}

$\therefore$ P(A) = {f, {1}, {2}, {3}, {1, 2}, {2, 3}, {3, 1}, {1, 2, 3}}.

✶ Venn Diagrams

A Swiss Mathematician Euler gave an idea to represent a set by the points in a closed curve. Later on

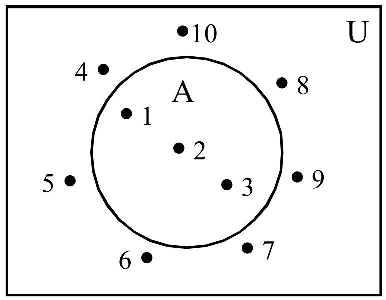
British Mathematician John Venn (1834-1923) brought this idea to practice. So, the diagrams drawn to represent sets are called Venn Euler diagrams or simply Venn diagrams.

In venn diagrams, the universal et is represented by a rectangular region and a set is represented by circle or a closed geometrical figure inside the universal set. Also, an element of a set A is represented by a point within the circle of set A.

eg, If $U = \{1, 2, 3, 4, \ldots\ldots\ldots\ldots, 10\}$

and $A = \{1, 2, 3\}$

Then its venn diagram is as shown in the figure.



✷ Operations on Sets

Now, we introduce some operations on sets to construct new sets from the given ones.

1. Union of Sets

Let A and B be two sets, then union of A and B is a set of all those elements wich are in A or in B or in both A and B. It is denoted by $A \cup B$ and read as "A union B".

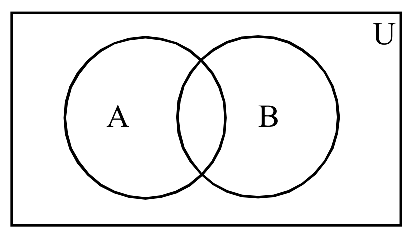
Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Clearly, $x \in A \cup B$

$\Rightarrow$ $x \in A$ or $x \in B$

If $x \notin A \cup B$

$\Rightarrow$ $x \notin A$ or $x \notin B$



$A \cup B$

The venn diagram of $A \cup B$ is as shown in the figure and the shaded protion represents $A \cup B$.

eg, If $A = \{1, 2, 3, 4\}$

and $B = \{4, 8, 5, 6\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$.

General Form

The union of a finite number of sets $A_1, A_2, \ldots\ldots\ldots\ldots, A_n$ is represented by

$$A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots\ldots \cup A_n \text{ or } \bigcup_{i=1}^{n} A_i$$

Symbolically, $\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for atleast one } i\}$

2. Intersection of sets

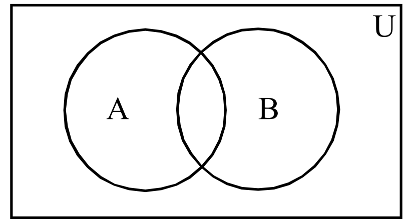
If A and B are two sets, then intersection of A and b is set of all those elements which are in both A and B. The intersection of A and B denoted by $A \cap B$ and read as "A intersection B".

Symbolically,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

If $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

and if $x \notin A \cap B \Rightarrow x \notin A$ and $x \notin B$



$A \cap B$

The venn diagram of $A \cap B$ is as shown in the figure and the shaded region represents $A \cap B$.

eg, If $A = \{1, 2, 3, 4\}$ and $B = \{4, 3, 5, 6\}$

$\therefore$ $A \cap B = \{3, 4\}$

General Form

The intersection of a finite number of sets A1, A2, A3,.................,An is represented by

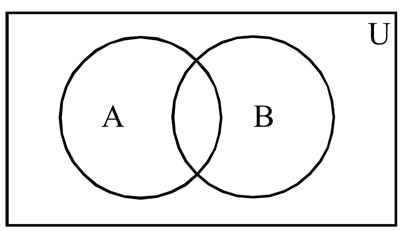
$$A_1 \cap A_2 \cap A_3 \cap \ldots\ldots\ldots\ldots \cap A_n \text{ or } \bigcup_{i=1}^{n} A_i$$

Symbolically, $\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for all } i\}$

3. Disjoint Sets

Two sets A and B are known as disjoint sets, if A B = ϕ ie, if A and B have no common element.

The venn diagram of disjoint sets as shown in the figure.



$A \cap B = \phi$

eg, if $A = \{1, 2, 3\}$

and $B = \{4\ 5, 6\} \Rightarrow A B = \{\ \} = \phi$

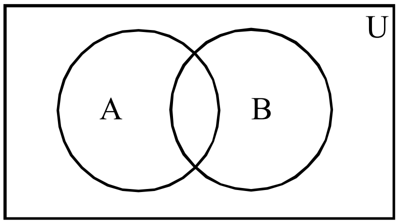
$\therefore$ A and B are disjoint sets.

4. Difference of Sets

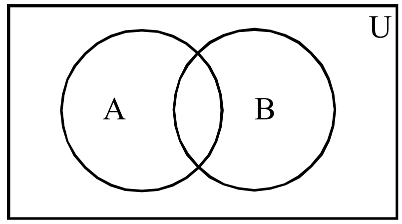
If A and B are two non-empty sets, then difference of A and B is a set of all those elements which are in A but not in B. It is denoted as $A - B$. If difference of two sets is $B - A$, then it is a set of those elements Which are in B but not in A.

Hence, $A - B = \{x : x \in A \text{ and } x \notin B\}$

and $B - A = \{x : x \in B \text{ and } x \notin A\}$



A – B



B – A

If $x \in$ A – B = $x \in$ A but $x \notin$ B

and if $x \in$ B – A = $x \in$ B but $x \notin$ A

The venn diagram of A – B and B – A are as shown in the figure and shaded region represents A – B and B – A.

eg, If A = {1, 2, 3, 4}

and B = {4, 5, 6, 7, 8}

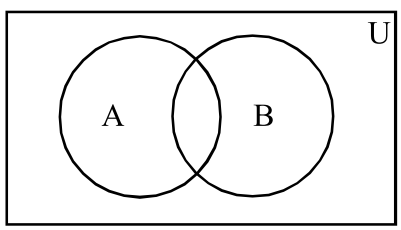
A – B = {1, 2, 3}

B – A = {5, 6, 7, 8}

5. Symmetric Difference of Sets

If A and are two sets, then set (A – B) ∪ (B – A) is known as symmetric difference of sets A and B and is denoted by A Δ B.

The venn diagram of A Δ B is as shown in the figure and shaded region represents A Δ B.



eg, A = {1, 2, 3} and B = {4, 5, 6}

then A Δ B = (A – B) (B – A)

= {1, 2 , 3} ∪ {4, 5, 6}

= {1, 2, 3, 4, 5, 6}

6. Complement of a Sets

The complement of a set A is the set of all those elements which are in universal set but not in A. It is denoted by A' or A^c.

If U is a universl set and A ⊂ U.

Then A' = U – A = {x : $x \in$ U but $x \notin$ A}

⇒ $x \in$ A ⇒ $x \notin$ A

The venn diagram of complement of a set A is as shown in the figure and shaded protion represents A'.

eg, If $U = \{1, 2, 3, 4, 5, \ldots\ldots\ldots\}$

and $A = \{2, 4, 6, 8, \ldots\ldots\ldots\}$

$\therefore$ $A' = U - A = \{1, 3, 5, 7, \ldots\ldots\ldots\}$

✷ Cardinal Number of a Finite set.

The number of distinct elements in a finite set A is called cardinal number and it is denoted by $n(A)$.

eg, If $A = \{-3, -1, 8, 10, 13, 17\}$

then $n(A) = 6$

Cardinal Number of Some Sets

If A, B nd C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) If A and B are disjoint sets. Then

$$n(A \cup B) = n(A) + (B)$$

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(iv) $n(A - B) = n(A) - n(A \cap B)$

(v) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(vi) $n(A') = n(U) - n(A)$

(vii) $n(A' \cup B') = n(U) - n(A \cap B)$

(viii) $n(A' \cap B') = n(U) - n(A \cup B)$

(ix) $n(A \cap B') = n(A) - n(A \cap B)$

Note :

- The order of elements in a set has no importance eg, {1,2,3} and {3,2,1} are same sets.
- The repetition of elements in a set does not effect the set, eg, {1,2,3} and {1,1,2,3} both are same sets.
- If ϕ represents a null set, then ϕ is never written with in braces ie, $\{\phi\}$ is not the null set.
- Equal set are equivalent sets while its converse need not to be true.
- Null set is a subset of each set.
- Each set is a subset of itself.
- If A hsa n elements, then number of subsets of set A is 2^n.
- If A has n elements, then number of proper subsets is $2^n - 1$.
- Each element of a power set is a set.
- Power set of any set is always non-empty.
- If set A has n elements, then P(A) has 2^n elements.
- $A - B = B - A$
- $A - B \subseteq A$ and $B - A \subseteq B$
- $A - \phi = A$ and $A - A = \phi$
- The sets $A - B$, $B - A$ and $A \cap B$ are disoint sets.
- Symmetric difference can also be written as

$$A \Delta B = (A \cup B) - (A \cap B)$$

- $A\Delta B = B\Delta A$ (commutative)
- $\phi = U'$
- $\phi' = U$
- $(A')' = A$
- $A \cup A' = U$
- $A \cap A' = \phi$